Comparison of Two Types of Density-Tapered Superscale Antenna Array

Jiahao Tian[1,a], Yongsheng Zhang[1,b,*], Jianxiong Zhou[1,c], Yang Huang[1,d], Qingfeng Tan[1,e] and Ji Chen[1,f]

[1]College of Electronic Science and Technology National University of Defense Technology, Changsha, 410073, China

a. tianjiahao@nudt.edu.cn, b. yszhang2001@vip.163.com, c. zjxjanet@sina.com, d. huangyang@nudt.edu.cn, e. 17678313758@163.com, f. 1141200969@qq.com

**Yongsheng Zhang*

Keywords: Massively thinned arrays, array antenna, sidelobe control, density-tapered antenna array.

Abstract: Sparse array can reduce the construction cost and complexity when the array size becomes large. In this paper, two types of ultra-large sparse planar antenna arrays are designed using density-tapered method. They are respectively the sparse array whose array element positions are selected at the grid point and the sparse array whose array element positions are randomly placed within the aperture range. By comparing with the uniform random plane antenna array, several important conclusions about designing the array by density tapering method are obtained. It is concluded that the performance of non-uniform density weighted array is better than that of uniform random array. The effect of array element number and sparsity on array performance is also analyzed.

1. Introduction

Reducing the sidelobe of antenna array pattern is an important problem in antenna array design. For small and medium-sized antenna arrays, satisfactory results can be obtained by using genetic algorithm, particle swarm optimization algorithm and simulated annealing algorithm. However, for a very large antenna array, because the number of variables to be optimized increases sharply, the array layout using the above method will encounter problems such as large amount of calculation, long consumption time and easy to fall into local optimal results. Therefore, the density-tapered method is used to reduce the sidelobe of the VLA. Each antenna of the density-tapered array transmits the same power as the transmitting array element and adopts the same amplitude weighting as the receiving array element. Density-tapered antenna array simulates the amplitude weighting of uniform full array by density weighting of array elements in position, so the array element spacing of density-tapered antenna array is not equal, and the average array element spacing near the center of the array is smaller, while the average array element spacing far from the center of the array is larger.

In this paper, the sparse array whose element's positions are limited to raster points and the sparse array whose elements can be randomly placed within the aperture range are designed by density-tapered method. Firstly, the angular resolution and sidelobe level of the density-tapered antenna arrays were analyzed. Then, by changing the number of array elements, the influence of the elements number on the angular resolution and the level of sidelobe of the antenna array is analyzed. Finally, the effect of sparsity on the performance of planar antenna array is studied under the condition that the number of array elements is not changed. At the end of this paper, several important conclusions about designing arrays with density-tapered method are summarized, which provides a reference for designing very large scale planar antenna arrays.

2. Problem Formulation

Suppose that the array works in the far-field and narrow-band conditions and that each antenna array element is the omnidirectional radiating antenna. The geometric model of uniform full-array is shown in Figure 1.

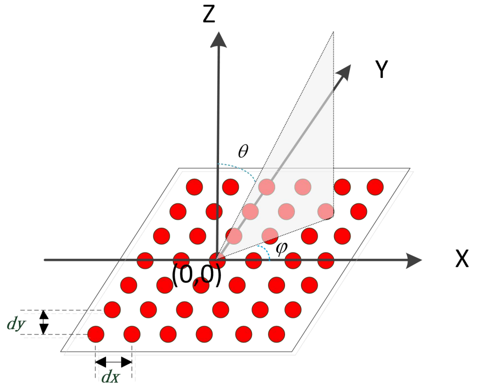


Figure 1: Geometric model of uniform full-array.

Array elements are placed in the X-O-Y plane. The angle between the direction of the signal source and the normal direction of the antenna plane and the positive direction of the x-axis is θ and Φ, respectively. The pattern of the array can be expressed as:

$$P(\theta,\varphi)=\sum_{m=1}^{M}\sum_{n=1}^{N}\exp\left[-j\frac{2\pi}{\lambda}\left(md_x u+nd_y v\right)\right] \quad (1)$$

Where $u=\sin\theta\cos\varphi-\sin\theta_0\cos\varphi_0$, θ is the pitch Angle of the array, φ is the azimuth Angle of the array , and $v=\sin\theta\sin\varphi-\sin\theta_0\sin\varphi_0$ If there is no special statement, θ 0= Φ 0=0.

In this paper, thinned planar arrays are divided into two types, one of which is called selected sparse arrays (SSA) whose element's positions are limited to raster points, and the other is called randomly placed sparse arrays (RSA) whose array element positions are randomly placed within the aperture range,. The difference between these two sparse arrays is vividly illustrated in Figure 2.

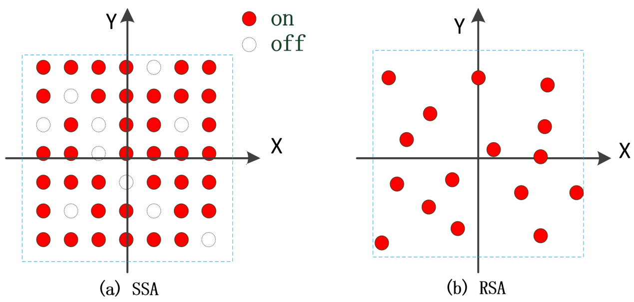


Figure 2: Geometric model of SSA and RSA.

The different methods to design these two kinds of arrays will be introduced in the third part of this paper.

3. Design SSA and RSA

3.1. Generation of SSA

The SSA can be generated by selecting elements from uniform full-array whose spacing between array elements is half of the wavelength. As described in Figure 2, when the array element is selected, the corresponding position is set to "on"; when the array element is not selected, the corresponding position is set to "off" . Suppose that the control variable of the array element is S, and the weighted normalized amplitude of the uniform full-array is A. If the size of array is (M×0.5λ) × (N×0.5λ) m2, S can be expressed as:

$$S(m,n)=\begin{cases}1 & rand < A(m,n)*k \\ 0 & rand \geq A(m,n)*k\end{cases} \tag{2}$$

Where rand is a random number evenly distributed between 0 and 1, and k is used to control the sparsity of the array. For a particular amplitude weighting, the sparsity can be calculated by following formula:

$$\eta_S=\frac{k\times\sum_{m=1}^{M}\sum_{n=1}^{N}A(m,n)}{MN} \tag{3}$$

For example, suppose the amplitude of the uniform full-array is weighted by:

$$A(m,n)=\cos^2\left(\frac{m-0.5M}{M}\times\pi\right)\cos^2\left(\frac{n-0.5N}{N}\times\pi\right) \tag{4}$$

and the sparsity is set to 0.2, the result of k is k = 4×ηs =0.8

The pattern of SSA can be expressed as:

$$P_S(u,v)=\sum_{m=1}^{M}\sum_{n=1}^{N}S(m,n)\exp\left[-j\frac{2\pi}{\lambda}\left(md_xu+nd_yv\right)\right] \tag{5}$$

Where 1 ≤ u ≤ 1 and 1 ≤ v ≤ 1.

3.2. Generation of RSA

Random placement doesn't mean setting elements at liberty. The position of sparse array elements is a random variable, and its distribution obeys a specific probability density function. Generating random numbers with probability density function to f(x) that x is in the range of a to b by 0-1 uniform distribution can be divided into three steps:

- Calculate the indefinite integral F(x).
- Generate the uniformly distributed random number y that ranges from F(a) to F(b).
- Solve F(x) = y to get the value of x.

For example, if f (x) = cos2 (π x / 2), -1 ⩽ x ⩽ 1, and the aperture of the array is L × L, the position of the m-th (m = 1, 2, • • • , MR) element on the x-axis is obtained by the following equations:

$$\frac{x_{temp}}{2}+\frac{\sin\left(\pi x_{temp}\right)}{2\pi}=rand_m-\frac{1}{2} \tag{6}$$

$$x_m=x_{temp}\times\frac{L}{2} \tag{7}$$

The determination of the position of this element on the y-axis is similar to that on the x-axis. The formula for calculating the sparsity of RSA is:

$$\eta_R = \frac{M_R \lambda^2}{4L^2} \tag{8}$$

Where MR is the total number of elements in planar sparse array.

The pattern of SSA can be expressed as:

$$P_R(u,v) = \sum_{m_R=1}^{M_R} \exp\left[-j\frac{2\pi}{\lambda}(x_m u + y_m v)\right] \tag{9}$$

4. Experiment and Analysis

In this section, the half power beamwidth (HPBW) and the highest peak sidelobe level (PSLL) of SSA and RSA are obtained by numerical simulation. The number of array elements changes from 256 to 16384, and the sparsity of the array changes from 0.25 to 0.0001. The wavelength used is λ=0.05m. FFT technology is used to accelerate the computation speed when calculating the pattern of SSA. When analyzing the performance of the array, the maximum value of two cross sections that φ = 0 and φ = 90 are used. In the simulation experiment, two density weighting functions are used to generate SSA and RSA. One is cos2 (x) function and the other is uniform random function (URF).

By changing the number of elements and calculating the PSLL of the pattern, we can observe the effect of the number of elements on the PSLL, as shown in Figure 3. The PSLL of SSA and RSA generated by uniform random function does not change obviously with the number of array elements. The PSLL of SSA and RSA generated by cos2(x) function decreases with the increase of the number of array elements. This shows that the non-uniform density weighting method can improve the performance of sparse array.

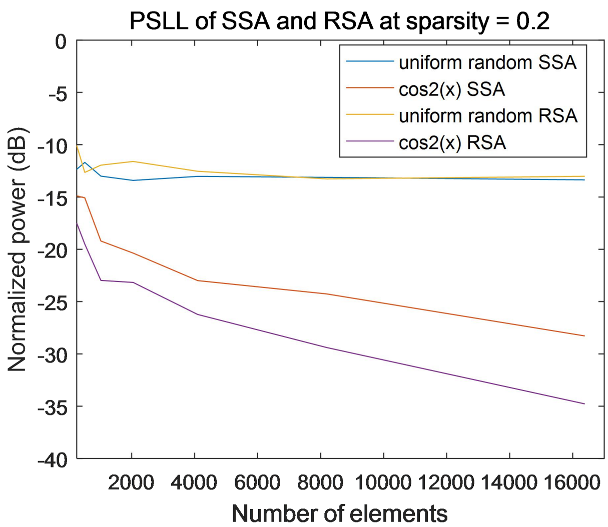


Figure 3: Effect of element number on PSLL.

When the sparsity of SSA and RSA is constant, change the number of array elements and calculate the HPBW of SSA and RSA, we can see how HPBW changes with the number of array elements, as shown in Figure 4.

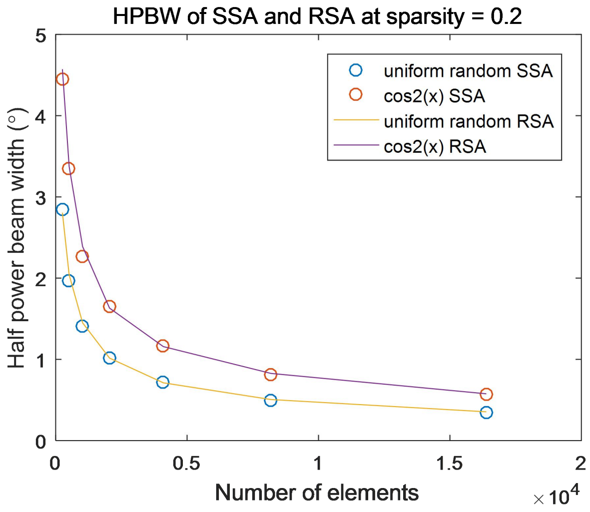


Figure 4: Effect of element number on HPBW.

It can be seen that the HPBW of SSA and RSA generated by uniform random function is smaller than that generated by cos2 (x) density weighting function. With the increase of the number of array elements, the aperture of the array becomes larger, and the directional resolution ability of the array is improved.

In order to study the change of PSLL and HPBW with sparsity when the number of elements is fixed, the number of elements of the thinned sparse array is set to 1024, and the sparsity changes from 0.25 to 0.0001. It should be noted that the SSA generated by the density-tapered method is random, and the number of elements is not strictly equal to 1024. The element's number of RSA is strictly equal to 1024. The results are summarized in table 1. The positions of the elements are generated by the density weighting function cos2 (x).

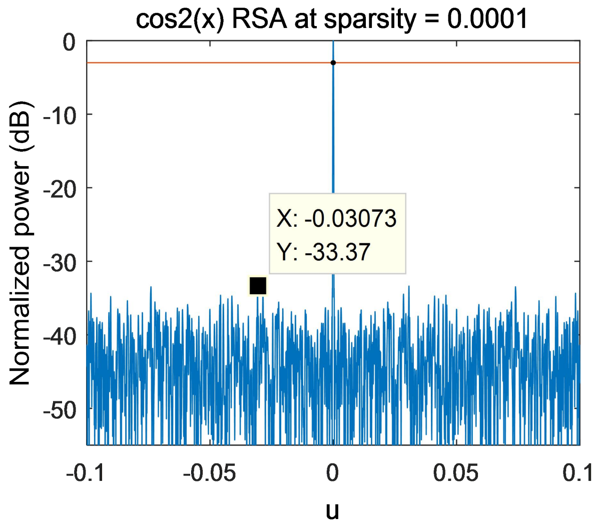
Table 1: Pattern index of SSA and RSA with element number of 1024.

Performance index	**Sparsity of density-tapered array**						
	0.25	***0.2***	***0.15***	***0.1***	***0.01***	***0.001***	***0.0001***
PSLL of SSA (dB)	-19.89	-19.21	-18.76	-19.06	-18.23	-16.41	-15.29
PSLL of RSA (dB)	-23.46	-23.0	-23.46	-22.31	-22.31	-21.98	-21.98
HPBW of SSA ( )	2.582	2.266	2.054	1.664	0.5213	0.1705	0.0486
HPBW of RSA ( )	2.688	2.389	2.072	1.703	0.5344	0.1684	0.0526

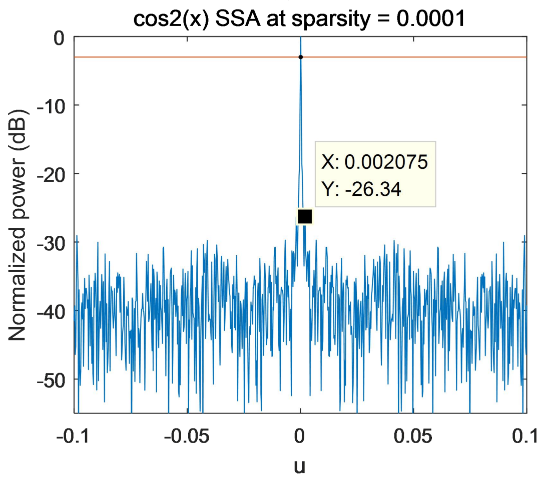
It can be seen that with the decrease of sparsity, the PSLL of SSA and RSA will be higher and higher. This indicates that if we want to keep the performance of the sparse array, the aperture of the array cannot be expanded without limitation. The maximum array arrangement can be obtained by density-tapered method.

The density-tapered method is suitable for designing ultra large planar antenna arrays, which has been used to design SSA with hundreds of elements. In this paper, SSA and RSA with 16384 elements are simulated and analyzed, and good results are achieved. The beam patterns of SSA and RSA with 16348 elements whose sparsity is 0.0001 are shown in Figure 5.

It can be seen that the psll of RSA is lower than that of SSA. When the number of array elements becomes large, the psll of the array will become very low, although the array has become very sparse.



(a)



(b)

Figure 5: Beam patterns of SSA and RSA with 16348 elements.

5. Conclusions

By analyzing the data in the fourth part, some meaningful conclusions can be obtained. Although these data are not obtained by the average value after many experiments, the probability of extreme cases is very small because of the large number of array elements.

- The position of RSA has no space limitation, and can better simulate the amplitude weighting of full array, so the PSLL performance of RSA is better than that of SSA.
- With the same sparsity, the more actual number of elements is, the lower the PSLL will be. However, the PSLL of uniform density weighted sparse array is basically maintained at a certain level. It is shown that the performance of sparse array with uniform random distribution of element position is inferior to that of non-uniform density weighted array.
- With the same number of array elements, as the array becomes thinner, the performance of the PSLL of the cos2(x) density weighted sparse array tends to deteriorate on the whole. Therefore,

the maximum aperture array with a fixed number of array elements can be found, which can provide a reference for the actual construction.

- The angular resolution of non-uniformly density taper array is not as good as that of uniform random array. And with the increase of aperture, the antenna array can distinguish targets with smaller angle difference.

Acknowledgments

This work was supported by the National Natural Science Foundation of China under Grant numbers 61771478.

References

[1] B. V. Ha, M. Mussetta, P. Pirinoli and R. E. Zich, "Modified Compact Genetic Algorithm for Thinned Array Synthesis," in IEEE Antennas and Wireless Propagation Letters, vol. 15, pp. 1105-1108, 2016.

[2] Y. Lo, "A mathematical theory of antenna arrays with randomly spaced elements," in IEEE Transactions on Antennas and Propagation, vol. 12, no. 3, pp. 257-268, May 1964.

[3] M. Skolnik, J. Sherman and F. Ogg, "Statistically designed density-tapered arrays," in IEEE Transactions on Antennas and Propagation, vol. 12, no. 4, pp. 408-417, July 1964.

[4] B. Steinberg, "The peak sidelobe of the phased array having randomly located elements," in IEEE Transactions on Antennas and Propagation, vol. 20, no. 2, pp. 129-136, March 1972.

[5] X. Wang, Y. Jiao and Y. Tan, "Synthesis of Large Thinned Planar Arrays Using a Modified Iterative Fourier Technique," in IEEE Transactions on Antennas and Propagation, vol. 62, no. 4, pp. 1564-1571, April 2014.